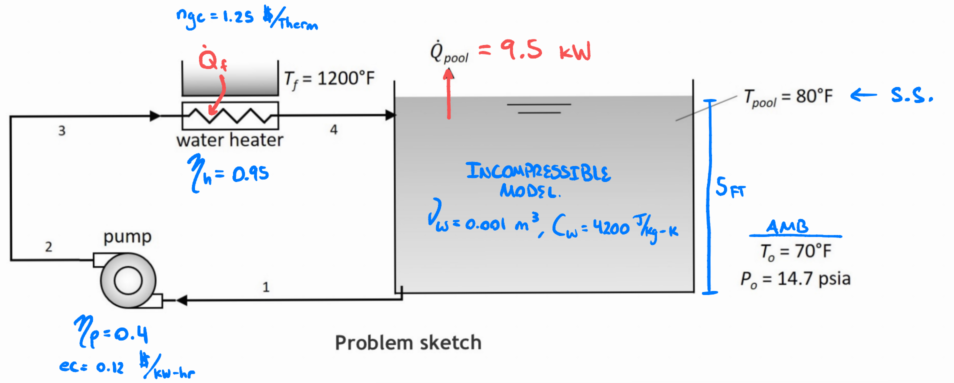
TYLER STEVENS
ME 461 - CHEADLE

JUNE 24, 2021

DESIGN [A]

Swimming Pool Heating System → SETTING UP THE PROBLEM

IGNORE ΔP IN PIPES



Problem sketch

PUMP CURVE (LINEAR)

$$\dot{V}_{oc} = 5\times10^{-7}\left[\frac{m^3\text{-}min}{s\text{-}rev}\right]N$$

$$\Delta P = \Delta P_{dh}\left(1 - \frac{\dot{V}}{\dot{V}_{oc}}\right) \qquad \Delta P_{dh} = 150\left[\frac{Pa\text{-}Min}{rev}\right]N$$

WATER HEATER SYSTEM CURVE

$$\Delta P_h = 3\times10^{12}\left[\frac{Pa\text{-}s}{m^6}\right]\dot{V}^2$$

FIXING THE STATES + PERFORMANCE BALANCES

[STATE 4-1] POOL

MASS BALANCE $I = 0 \quad \therefore \quad \dot{m}_1 = \dot{m}_4 = \dot{m}$ → UNIFORM MASS FLOWRATE THROUGHOUT $\quad \dot{m} = \dot{V}/v$ ← CONST.

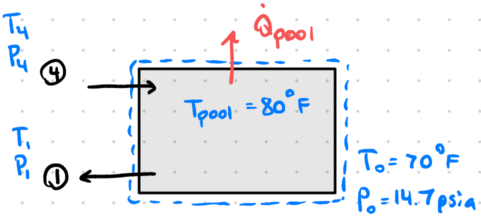
ENERGY BALANCE $I = 0$

$$\dot{m}h_4 = \dot{Q}_{pool} + \dot{m}h_1$$

ENTROPY BALANCE $I + G = 0 + \cancel{S}^{0} + \cancel{S}^{S.S}$

$$\dot{S}_{gen_{pool}} = \frac{\dot{Q}_{pool}}{T_{pool}} + \dot{m}(s_4 - s_1) \longrightarrow s_4 - s_1 = c_w \ln\left(\frac{T_4}{T_1}\right)$$

FOR INC. C_w = CONST.



[STATE 1] PUMP INLET

$u_1 = C_w T_1$ INC. MODEL

$h_1 = C_w T_1 - v P_1$ @ $T_{REF} = 0,\ P_{REF} = 0$

$T_1 = T_{pool} = 80°F$

$P_1 \approx P_o = 14.7$ psia ← ASSUMPTION FOR P_2 = AMBIENT.

STATE FIXED ✓

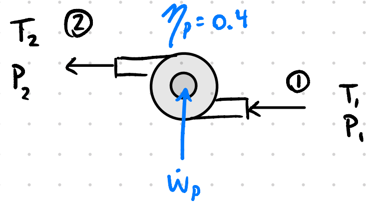
RECALL) $(h_2 - h_1) = (u_2 - u_1) + v(P_2 - P_1)$

$h = u + Pv$

$(u_2 - u_1) = c(T_2 - T_1)$

Made with Goodnotes

[STATE 1-2] WATER PUMP



MASS BALANCE $I + \cancel{G}^{0} = 0 + \cancel{D}^{0} + \cancel{S}^{S.S.}$

$\boxed{\dot m_2 = \dot m_1} = \dot m = \dot\forall / \nu$

ENERGY BALANCE $I + \cancel{G}^{0} = 0 + \cancel{D}^{0} + \cancel{S}^{S.S.}$

$\dot m_1 h_1 + \dot W_p = \dot m_2 h_2$

ENTROPY BALANCE $I + G = 0 + \cancel{D}^{0} + \cancel{S}^{S.S.}$

$\eta_p = 0.4 = \frac{\dot W_{p,s}}{\dot W_p}$ ← WANT ← COST

$\dot m_1 s_1 + \dot S_{gen_p} = \dot m_2 s_2 \rightarrow \dot S_{gen_p} = \dot m (s_2 - s_1)$

↳ AND $(s_2 - s_1) = c_w \ln\left(T_2/T_1\right)$ ← FOR CONST c_w.

REVERSIBLE ENERGY BALANCE

$\dot m_1 h_1 + \dot W_{p,s} = \dot m_2 h_{2,s}$

$h_{2,s} = \underbrace{c_w T_1}_{} + \nu P_2$ → FROM $\Delta S = 0 = c_w \ln\left(T_2/T_1\right)$

↳ T_2 MUST $\equiv T_1$

[STATE 2] PUMP OUTLET

$P_2 = P_1 + \Delta P$ SEE PUMP CURVE.

↳ PRESSURE RISE PROVIDED BY PUMP (FUNC OF $\dot\forall$, N)

T_2 FROM E-BAL STATE 1-2

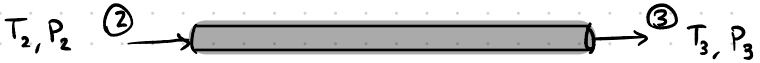
STATE FIXED ✓ h_2 ✓ u_2 ✓

$\Delta P = \Delta P_{sh}\left(1 - \frac{\dot\forall}{\dot\forall_{oc}}\right)$

[STATE 2-3] PIPING

MASS BALANCE $\dot m_3 = \dot m_2$ E BAL $\dot m_3 h_3 = \dot m_2 h_2$

$P_3 = P_2$
$T_3 = T_2$ } IGNORING LOSSED DUE TO PIPING/GEOMETRY/FRICTION



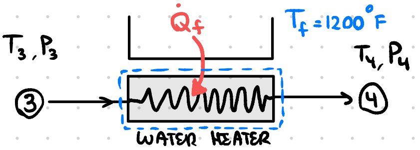
[STATE 3] WATER HEATER INLET

STATE FIXED ✓

$T_3 = T_2$, $P_3 = P_2$, $h_3 = h_2$, $u_3 = u_2$

$\eta_h = 0.95 = \frac{\dot Q_{f,s}}{\dot Q_f}$ ← WANT ← COST

[STATE 3-4] WATER HEATER



ENERGY BALANCE $I + \cancel{G}^{0} = 0 + \cancel{D}^{0} + \cancel{S}^{S.S.}$

$\dot m_3 h_3 + \dot Q_f = \dot m_4 h_4$ AND $h_4 = c_w T_4 + \nu_w P_4$

ENTROPY BALANCE $I + G = 0 + \cancel{D}^{0} + \cancel{S}^{S.S}$

$\dot m_3 s_3 + \frac{\dot Q_f}{T_f} + \dot S_{gen_h} = \dot m_4 s_4 \rightarrow \dot S_{gen_h} = \dot m (s_4 - s_3) - \frac{\dot Q_f}{T_f}$ AND $(s_4 - s_3) = c_w \ln\left(\frac{T_4}{T_3}\right)$

REVERSIBLE ENERGY BALANCE

$I + \cancel{G}^{0} = 0 + \cancel{D}^{0} + \cancel{S}^{0}$ ∴ $\dot m_3 h_3 + \dot Q_{f,s} = \dot m_4 h_{4,s}$

$h_{4,s} = c_w T_3 + \nu P_4$ → ISENTROPIC, $\Delta S = 0$ ∴ $\ln\left(T_4/T_3\right) = 0$

Made with Goodnotes

[STATE 4] HEATER OUTLET

$P_4 = P_3 - \Delta P_h$

$\Delta P_h = 3 \times 10^{12} \left[\frac{Pa-s}{m^6}\right] \dot{V}^2$

$u_4 = c_w T_4$

h_4 ✓

↳ HEATER PRESSURE DROM SYSTEM CURVE

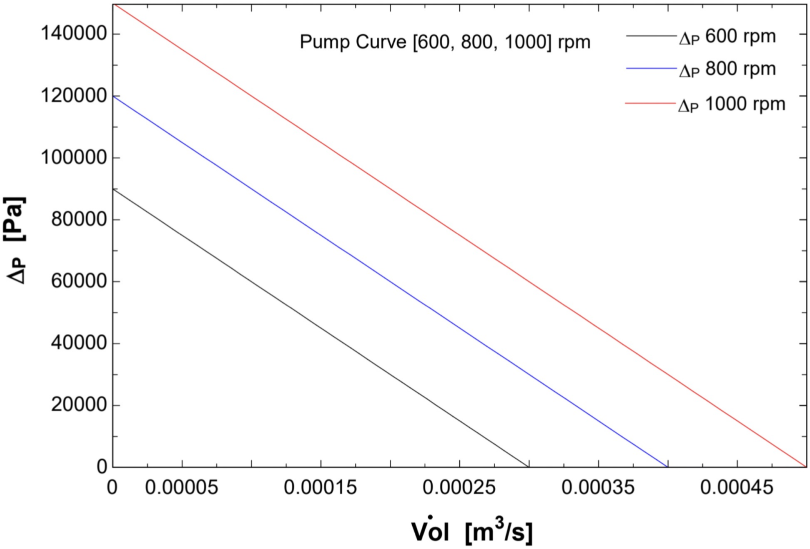
T_4 ✓ → TIED TO STATE 1 (ALL STATES CONNECTED ⇒ ABLE TO SOLVE)

ALL STATES FIXED ✓

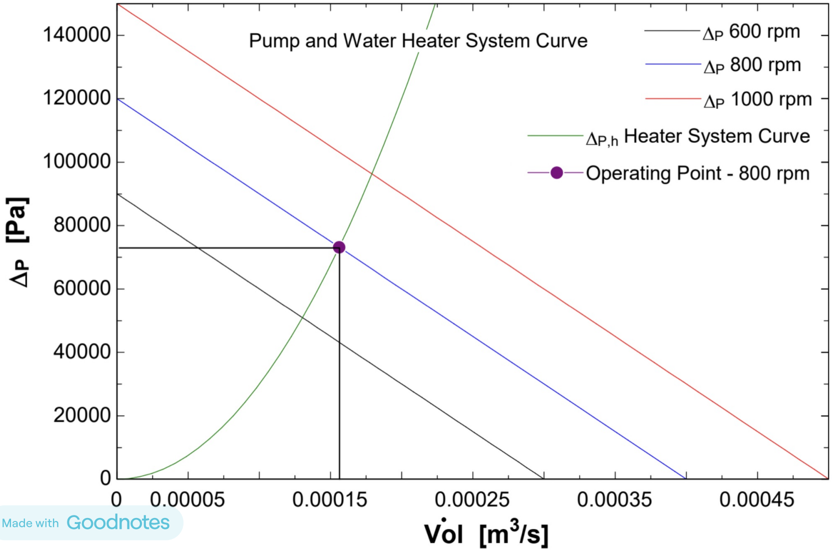
ⓐ PLOT THE PUMP CURVE @ 600, 800 + 1000 RPM

$\dot{V}_{oc}$ = MAX FLOW RATE, NO ΔP, ΔP_{sh} = MAX ΔP, NO FLOW RATE.

$$\begin{cases} \dot{V} = [0, \dot{V}_{oc}] \\ \Delta P = [0, \Delta P_{sh}] \end{cases}$$



ⓑ SYSTEM CURVE OVERLAY.



★ OPERATING POINT INDICATES MAX $\dot{V}$ POSSIBLE THAT STILL PROVIDES LARGE ENOUGH ΔP TO PUMP FLUID THROUGH THE HEATER (@ 800 rpm)

$\dot{V} = 0.000156155 \ m^3/s$

Made with Goodnotes

ⓒ DETERMINE $\dot{\forall}$ PUMPED THROUGH THE SYSTEM.

- SET $N = 800$ RPM
- SET $\Delta P = \Delta P_h$

↳ SOLVE FOR $\dot{\forall}$

NEED $\underset{\uparrow\ \text{PUMP}}{\Delta P} \geq \underset{\uparrow\ \text{PRESSURE LOST IN HEATER}}{\Delta P_h}$ TO DRIVE FLOW.

EES SOLUTION) $\dot{\forall} = 0.000156155\ m^3/s$ $\Delta P = 73{,}154$ Pa

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ⓓ DETERMINE PRESSURE AND TEMPERATURE @ ALL STATES.

↳ SOLVED THROUGH ALL STATE DEFINITIONS AND PERFORMANCE BALANCES ABOVE WITH $N = 800$ rpm, $\dot{\forall} = 0.000156155\ m^3/s$

STATE$_i$	T_i [K] {[F]}	P_i [Pa] {[atm]}
1	299.8 {80}	101353 {1}
2	299.9 {80.2}	174506 {1.722}
3	299.9 {80.2}	174506 {1.722}
4	314.3 {106.1}	101353 {1}

ⓔ DETERMINE THE PUMP POWER, $\dot{W}_p$

↳ FROM STATE 1-2: $\dot{W}_p = \dot{m}(h_2 - h_1)$ $\eta_p = 0.4 = \frac{\dot{W}_{p,s}}{\dot{W}_p}$

$\dot{W}_p = 28.56$ [W]

ⓕ DETERMINE $\dot{S}_{gen_p}$ AND $\dot{S}_{gen_h}$

$\dot{S}_{gen_p} = \dot{m}(s_2 - s_1)$ AND $(s_2 - s_1) = c_w \ln(T_2/T_1) \longrightarrow \dot{S}_{gen,p} = 0.2389$ [W/K]

$\dot{S}_{gen_h} = \dot{m}(s_4 - s_3) - \frac{\dot{Q}_f}{T_f}$ AND $(s_4 - s_3) = c_w \ln\left(\frac{T_4}{T_3}\right) \longrightarrow \dot{S}_{gen,h} = 20.43$ [W/K]

ⓖ DETERMINE THE TOTAL LOST WORK IN THE PUMP + HEATER COMBINED.

$\dot{S}_{gen_{PUMP,HEATER}} = \dot{S}_{gen_p} + \dot{S}_{gen_h}$ $\dot{W}_{LOST} = \dot{S}_{gen_{PUMP,HEATER}} \cdot \overset{\text{DEAD STATE}}{T_0} = 6083$ [W]

ⓗ DETERMINE THE TOTAL COST ($/hr) TO RUN THIS POOL SYSTEM.

ec = 0.12 * CONVERT($/kW-hr, $/W-hr) ngc = 1.25 * CONVERT($/therm, $/W-hr)

$Cost_p = ec * \dot{W}_p$

$Cost_h = ngc * \dot{Q}_f$

$Cost_{TOT} = \underset{0.003427\ \$/hr}{Cost_p} + \underset{0.404\ \$/hr}{Cost_h} \longrightarrow Cost_{TOT} = 0.4074$ ($/hr)

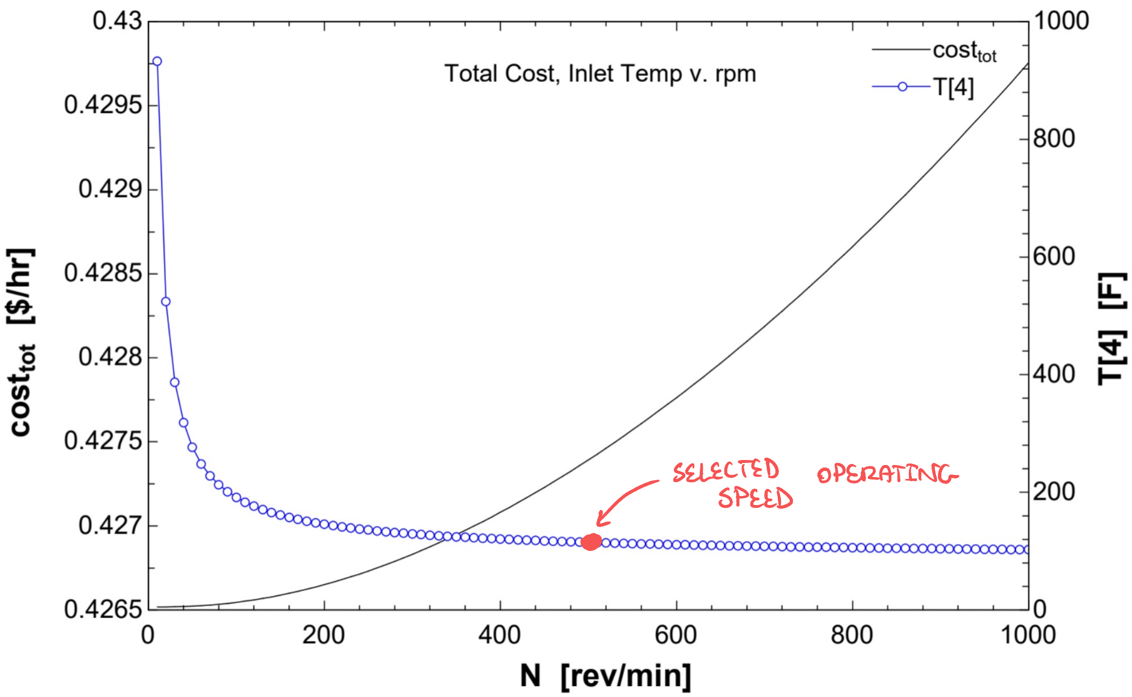
Made with Goodnotes

ⓘ FRACTION OF COST ASSOCIATED W/ RUNNING THE WATER HEATER?

$$Frac_{cost,h} = \frac{cost_h}{(cost_h + cost_p)} = \boxed{0.9916 \longrightarrow 99.2\% \text{ HEATING COST.}}$$

ⓙ PLOT OF TOTAL COST AND POOL INLET TEMP (T_4) V. RPM

↳ PARAMETRIC SWEEP N. ↳ SET $\Delta P = \Delta P_h$ ↳ SOLVE FOR $\dot{V}$ EACH RUN.



SELECTED OPERATING SPEED

OPERATING SPEED SELECTION

↳ N = 500 RPM AS YOU CAN SEE ON THE PLOT, COST INCREASES NEGLIGIBLY WITHIN THE SPEED RANGE. HIGHER SPEEDS REDUCE POOL INLET TEMPERATURES.

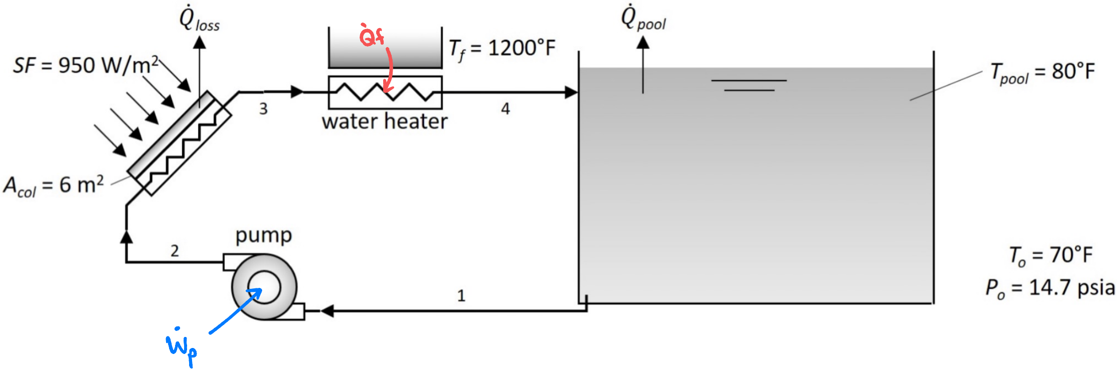
I SELECTED THIS OPERATING POINT AS A MIDDLE GROUND BETWEEN COST AND INLET TEMP.

(TOTAL COST /YEAR ~ $50 DIFFERENCE BETWEEN LOW AND HIGH SPEED. → NEGLIGIBLE)

Made with Goodnotes

MODIFIED POOL HEATING SYSTEM (SOLAR COLLECTOR)

DESIGN [B]



SOLAR COLLECTOR MODEL

SF = 950 W/m² (SOLAR FLUX)

$A_{col} = 6\,m^2$

$\dot{Q}_{loss} = UA\,(T_3 - T_o)$

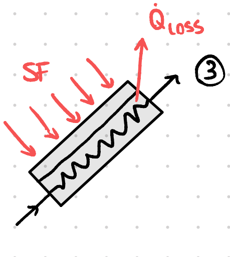
↳ $UA = 120 \quad W/K$

SOLAR COLLECTOR SYSTEM CURVE

$$\Delta P_{col} = 5\times10^{12} \left[\frac{Pa - s^2}{m^6}\right] \dot{\forall}^2$$

NEW PERFORMANCE BALANCES AND FIXED STATES

STATE [2-3] SOLAR FLUX COLLECTOR



MASS BALANCE

$$I + \cancelto{0}{G} = O + \cancelto{0}{D} + \cancelto{S.S.}{S}$$

$\dot{m}_2 = \dot{m}_3 \Rightarrow \dot{m}$

ENERGY BALANCE

$$I + \cancelto{0}{G} = O + \cancel{D} + \cancelto{S.S.}{S}$$

$$\dot{m}h_2 + \dot{W}_{SF} = \dot{Q}_{Loss} + \dot{m}h_3$$

ENTROPY BALANCE

$$I + G = O + \cancelto{0}{D} + \cancelto{S.S.}{S}$$

$$\dot{m}s_2 + \dot{S}_{gen_{SF}} = \dot{m}s_3 + \frac{\dot{Q}_{Loss}}{T_3} \longrightarrow \dot{S}_{gen_{SF}} = \frac{\dot{Q}_{Loss}}{T_3} + \dot{m}\,(s_3 - s_2)$$

↳ $= c_w \ln\left(\frac{T_3}{T_2}\right)$

$SF = 950\,W/m^2$

$A_{col} = 6\,m^2$

$\therefore \dot{W}_{SF} = 950 \cdot 6 \; [W]$

↳ $\dot{W}_{SF} = 5700 \; [W]$

$h_3 = c_w T_3 + \nu_w P_3$

$\dot{Q}_{Loss} = UA\,(T_3 - T_o)$

[STATE 3] SOLAR COLLECTOR OUTLET

$P_3 = P_2 - \Delta P_c$ → PRESSURE LOST IN SOLAR COLLECTOR (SYSTEM CURVE)

T_3 ✓

u_3 ✓

h_3 ✓

★ ALL OTHER STATES + BALANCES REMAIN THE SAME

Made with Goodnotes

ⓚ DETERMINE $\dot{\forall}$ @ $N = 800$ rpm.

- SET $N = 800$ RPM
- SET $\Delta P = \Delta P_h + \Delta P_{col}$

↳ SOLVE FOR $\dot{\forall}$

NEED $\underset{\text{PUMP}}{\Delta P} \geq \underset{\text{PRESSURE LOST IN HEATER}}{\Delta P_h} + \Delta P_{col}$ TO DRIVE FLOW.

PRESSURE LOST IN SOLAR COLLECTOR

EES SOLUTION) $\dot{\forall} = 0.0001051516 \ m^3/s$ $\Delta P = 88455 \ Pa$

ⓛ DETERMINE T, P FOR EACH STATE IN THE MODIFIED SYSTEM.

↳ SOLVED THROUGH ALL STATE DEFINITIONS AND PERFORMANCE BALANCES ABOVE WITH $N = 800$ rpm, $\dot{\forall} = 0.0001051516 \ m^3/s$

STATE i	T_i [K] {[F]}	P_i [Pa] {[atm]}
1	299.8 {80}	101353 {1}
2	299.9 {80.22}	189807 {1.873}
3	308.8 {96.19}	134523 {1.328}
4	321.3 {118.7}	101353 {1}

ⓜ DETERMINE THE PUMP POWER, $\dot{W}_p$

$$\dot{W}_p = \dot{m}(h_2 - h_1) \qquad \eta_p = 0.4 = \frac{\dot{W}_{p,s}}{\dot{W}_p}$$

$\dot{W}_p = 23.25$ [W]

ⓝ FRACTIONS OF TOTAL POOL HEATING LOAD DELIVERED BY THE WATER HEATER AND SOLAR COLLECTOR. WHY DONT THEY ADD TO 1?

$$\text{FRAC}_{\text{LOAD}\,h} = \frac{\dot{Q}_f}{\dot{Q}_{pool}} = 0.5814 \approx 58.1\%$$

← WATER HEATER

$$\text{FRAC}_{\text{LOAD}\,sc} = \frac{\dot{W}_{sc} - \dot{Q}_{loss}}{\dot{Q}_{pool}} = 0.4162 \approx 41.6\%$$

← SOLAR COLLECTOR

$0.4162 + 0.5814 = 0.9976$ ⟶ SLIGHTLY < 1 BECAUSE THE PUMP SLIGHLY HEATS THE FLUID AS IT INCREASES PRESSURE.

(WOULD BE > 1 IF DIDNT ACCOUNT FOR $\dot{Q}_{LOST}$ TO SURROUNDINGS FROM THE SOLAR COLLECTOR)

Made with Goodnotes

ⓞ ENTROPY GENERATED IN THE PUMP AND WATER HEATER

$\dot{S}_{gen_p} = \dot{m}\left(s_2 - s_1\right)$ AND $\left(s_2 - s_1\right) = c_w \ln\left(T_2/T_1\right) \longrightarrow \dot{S}_{gen,p} = 0.1796\ [W/K]$

$\dot{S}_{gen_h} = \dot{m}\left(s_4 - s_3\right) - \dfrac{\dot{Q}_f}{T_f}$ AND $\left(s_4 - s_3\right) = c_w \ln\left(\dfrac{T_4}{T_3}\right) \longrightarrow \dot{S}_{gen,h} = 11.55\ [W/K]$

ⓟ TOTAL LOST WORK IN PUMP + HEATER COMBINED.

DEAD STATE = T_0

$\dot{S}_{gen_{pump,\,heater}} = \dot{S}_{gen_p} + \dot{S}_{gen_h}$

$\dot{W}_{lost} = \dot{S}_{gen_{pump,\,heater}} \cdot T_0 = 3452\ [W]$

ⓠ TOTAL COST ($/hr) TO RUN THE MODIFIED POOL HEATING SYSTEM?

ec = 0.12 * CONVERT($/kW-hr, $/W-hr)

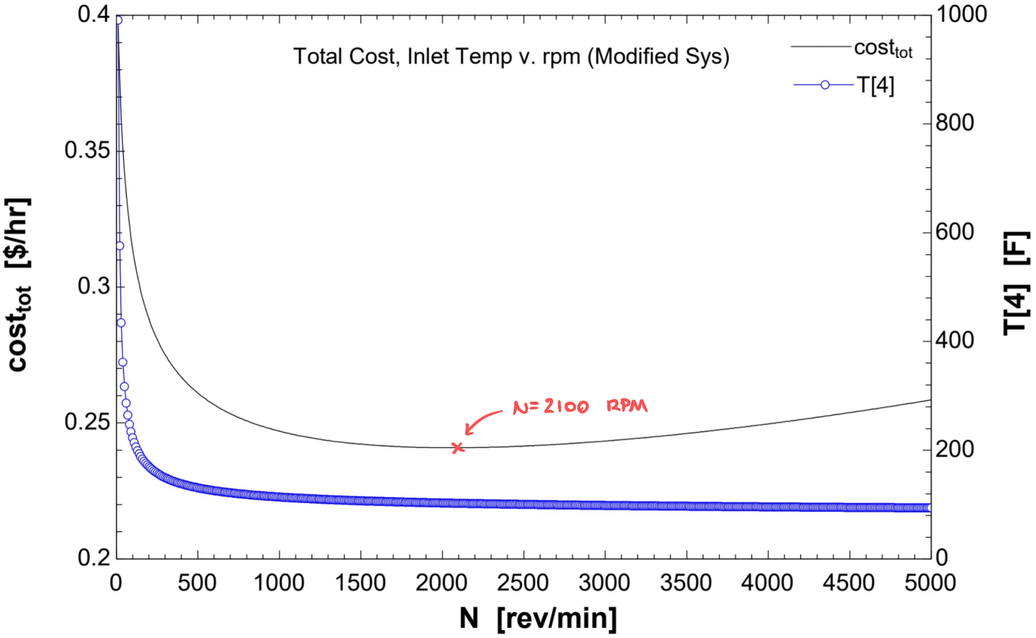
ngc = 1.25 * CONVERT($/therm, $/W-hr)

$Cost_p = ec * \dot{W}_p$

$Cost_h = ngc * \dot{Q}_f$

$Cost_{tot} = \underset{\substack{\| \\ 0.00279\ \$/hr}}{Cost_p} + \underset{\substack{\| \\ 0.2356\ \$/hr}}{Cost_h} \longrightarrow COST_{TOT} = 0.2384\ \$/hr$

ⓡ PLOT: TOTAL COST + INLET TEMP (T_4) v. RPM.



OPTIMAL PUMP SPEED EXISTS $N \approx 2100$ rpm. $COST_{TOT} = 0.2409\ \$/hr$

Made with Goodnotes

EES CODE APPENDIX - DESIGN A (NO SOLAR COLLECTOR)

```
// Tyler Stevens - ME 461 HW 3 - Cheadle - June 24, 2021
$UnitSystem SI K Pa J mass deg
$Tabstops 0.2 0.4 0.6 0.8 4

////////////////////////////////////////
//Pool Heating System [A]//
////////////////////////////////////////
DESIGN$ = 'A'

"knowns"
eta_p = 0.4 [-]  "pump effieciency"
eta_h = 0.95 [-]                                        "ng heater efficiency"
v_w = 0.001 [m^3/kg]                                    "spec vol of pool water"
c_w  =4200 [J/kg-K]                                     "spec heat capacity of water"
P_o = 14.7*convert(psi,Pa)
T_o = converttemp(F,K,70)
T_pool = converttemp(F,K,80)
T_f = converttemp(F,K,1200)
ec = 0.12*convert($/kW-hr,$/W-hr)
ngc = 1.25*convert($/therm,$/W-hr)

//////////////////////////////////////////////////////////////////////////////////
//Modified Pool Heating System - Solar Flux Collector [B]//
//////////////////////////////////////////////////////////////////////////////////
"knowns"
SF = 950 [W/m^2]
A_col = 6 [m^2]
UA = 120 [W/K]
W_dot_sc = SF*A_col

"Pump Curve"
DELTA_P_dh = 150[Pa-min/rev]*N                          "dead head pressure"
Vol_dot_oc = 5E-7[m^3-min/s-rev]*N                      "open circuit flow rate"
DELTA_P = DELTA_P_dh*(1 - (Vol_dot/Vol_dot_oc))         "pump curve"

"System Curves"
DELTA_P_h = 3E12[Pa-s^2/m^6]*Vol_dot^2                  "Water Heater System Curve"
DELTA_P_col = 5E12[Pa-s^2/m^6]*Vol_dot^2                "Solar Flux Collector System Curve"
DELTA_P_sys = DELTA_P_h + DELTA_P_col                   "Combined Systems Curve Pressure Loss"

"Fixed Parameters"
m_dot = Vol_dot/v_w
// mass balance
{mass flow rate constant for all states: m = m4 = m3 = m2 = m1}

"! State 4-1 Pool"
Q_dot_pool = 9.5*convert(kW,W)
// energy balance
m_dot*h[4] = Q_dot_pool + m_dot*h[1]
// entropy balance
S_dot_gen_pool = (Q_dot_pool/T_pool) + m_dot*c_w*ln(T[1]/T[4])

"! State 1 - Pump Inlet"
T[1] = T_pool
P[1] = P_o
u[1] = c_w*T[1]
h[1] = u[1] + v_w*P[1]

"! State 1-2 Water Pump"
eta_p=W_dot_ps/W_dot_p
// energy balance
m_dot*h[1] + W_dot_p = m_dot*h[2]
// entropy balance
DELTA_s_21 = c_w*ln(T[2]/T[1])
```

Made with Goodnotes

```
S_dot_gen_p = m_dot*DELTA_s_21
// reversible energy balance
m_dot*h[1] + W_dot_ps = m_dot*h_s[2]
h_s[2] = c_w*T[1] + v_w*P[2]

"! State 2 - Pump Outlet"
P[2] = P[1] + DELTA_P
h[2] = c_w*T[2] - v_w*P[2]
u[2] = c_w*T[2]

"! State 3-4 Water Heater"
eta_h = Q_dot_fs/Q_dot_f
// energy balance
m_dot*h[3] + Q_dot_f = m_dot*h[4]
h[4] = c_w*T[4] + v_w*P[4]
// entropy balance
S_dot_gen_h = m_dot*DELTA_s_43 - (Q_dot_f/T_f)
DELTA_s_43 = c_w*ln(T[4]/T[3])
// reversible energy balance
m_dot*h[3] + Q_dot_fs = m_dot*h_s[4]
//h_s[4] = c_w*T[3] + v_w*P[4]

"! State 4 - Heater Outlet"
P[4] = P[3] - DELTA_P_h
u[4] = c_w*T[4]

$IF DESIGN$ = 'A'
    "Part a - Pump Curve at different speeds"
    $IFNOT PARAMETRICTABLE
        Vol_dot = 0.000156155 [m^3/s]
        N = 800 [rev/min]                                   "pump speed"
        //DELTA_P = DELTA_P_h
    $ENDIF
    "Part c - Determine the volumetric flow rate of water throughout the system"
    $IF PARAMETRICTABLE
        DELTA_P = DELTA_P_h                                 "pressure differential req to overcome pressure
loss in heater"
    $ENDIF

    "! State 2-3 Piping"
    // mass balance
    {mass flow rate constant for all states: m3 = m2}

    "! State 3 - Heater Inlet"
    T[3] = T[2]
    P[3] = P[2]
    h[3] = h[2]
    u[3] = u[2]

    "Total Entropy Gen and Lost Work to heater and pump"
    S_dot_gen_hp = S_dot_gen_p + S_dot_gen_h
    W_dot_lost = S_dot_gen_hp*T_o
    "Total cost to run the pool heating system [$/hr]"
    cost_p = ec*W_dot_p
    cost_h = ngc*Q_dot_f
    cost_tot = cost_p + cost_h
    "Fraction of cost associated with running water heater"
    Frac_cost_h = cost_h/(cost_h+cost_p)
$ENDIF

$IF DESIGN$ = 'B'
    $IFNOT PARAMETRICTABLE
        Vol_dot = 0.0001051516 [m^3/s]
        N = 800 [rev/min]                                   "pump speed"
```

Made with Goodnotes

```
        //DELTA_P = DELTA_P_sys                                   "pressure differential req to overcome pressure
loss in heater"
    $ENDIF
    "Part c - Determine the volumetric flow rate of water throughout the system"
    $IF PARAMETRICTABLE
        DELTA_P = DELTA_P_sys                                     "pressure differential req to overcome pressure
loss in heater"
    $ENDIF

    "! State 2-3 Piping"
    // energy balance
    m_dot*h[2] + W_dot_sc = Q_dot_loss + m_dot*h[3]
    Q_dot_loss = UA*(T[3] - T_o)
    h[3] = c_w*T[3] + v_w*P[3]
    // entropy balance
    S_dot_gen_sf = Q_dot_loss/T[3] + (m_dot*(c_w*ln(T[3]/T[2])))

    "! State 3 - Heater Inlet"
    P[3] = P[2] - DELTA_P_col
    u[3] = c_w*T[3]

    "Fraction of Pool Heating Load delivered by water heater and solar collector"
    Frac_wh_load = Q_dot_f/Q_dot_pool
    Frac_sc_load = (W_dot_sc - Q_dot_loss)/Q_dot_pool

    "Total Entropy Gen and Lost Work to heater and pump"
    S_dot_gen_hp = S_dot_gen_p + S_dot_gen_h
    W_dot_lost = S_dot_gen_hp*T_o
    "Total cost to run the pool heating system [$/hr]"
    cost_p = ec*W_dot_p
    cost_h = ngc*Q_dot_f
    cost_tot = cost_p + cost_h
    "Fraction of cost associated with running water heater"
    Frac_cost_h = cost_h/(cost_h+cost_p)
$ENDIF
```

SOLUTION

Unit Settings: SI K Pa J mass deg

A_{col} = 6 [m^2]
$cost_h$ = 0.404 [$/hr]
$cost_p$ = 0.003427 [$/hr]
$cost_{tot}$ = 0.4074 [$/hr]
c_w = 4200 [J/kg-K]
Δ_P = 73154 [Pa]
$\Delta_{P,col}$ = 121922 [Pa]
$\Delta_{P,dh}$ = 120000 [Pa]
$\Delta_{P,h}$ = 73153 [Pa]
$\Delta_{P,sys}$ = 195075 [Pa]
$\Delta_{s,21}$ = 1.53 [J/kg-K]
$\Delta_{s,43}$ = 196.6 [J/kg-K]
DESIGN$ = 'A'
ec = 0.00012 [$/W-hr]
η_h = 0.95 [-]
η_p = 0.4 [-]
$Frac_{cost,h}$ = 0.9916 [-]
$\dot{m}$ = 0.1562 [kg/s]
N = 800 [rev/min]
ngc = 0.00004265 [$/W-hr]
P_o = 101353 [Pa]
$\dot{Q}_f$ = 9471 [W]
$\dot{Q}_{fs}$ = 8998 [W]
$\dot{Q}_{pool}$ = 9500 [W]
SF = 950 [W/m^2]
$\dot{S}_{gen,h}$ = 20.43 [W/K]
$\dot{S}_{gen,hp}$ = 20.67 [W/K]
$\dot{S}_{gen,p}$ = 0.2389 [W/K]
$\dot{S}_{gen,pool}$ = 0.7416 [W/K]
T_f = 922 [K]
T_o = 294.3 [K]
T_{pool} = 299.8 [K]
UA = 120 [W/K]
$\dot{Vol}$ = 0.0001562 [m^3/s]
$\dot{Vol}_{oc}$ = 0.0004 [m^3/s]
v_w = 0.001 [m^3/kg]
$\dot{W}_{lost}$ = 6083 [W] {6.083 [kW]}
$\dot{W}_p$ = 28.56 [W]
$\dot{W}_{ps}$ = 11.42 [W]
$\dot{W}_{sc}$ = 5700 [W]

No unit problems were detected.

SOLUTIONS DESIGNA (NO SOLAR)

KEY VARIABLES

$\dot{Vol}$ = 0.0001562 [m^3/s] *Part c/k) Volumetric Flow Rate of water pumped through the system.*
$\dot{W}_p$ = 28.56 [W] *Part e/m) Pump Power*
$\dot{S}_{gen,p}$ = 0.2389 [W/K] *Part f/o) Entropy generated in the pump.*

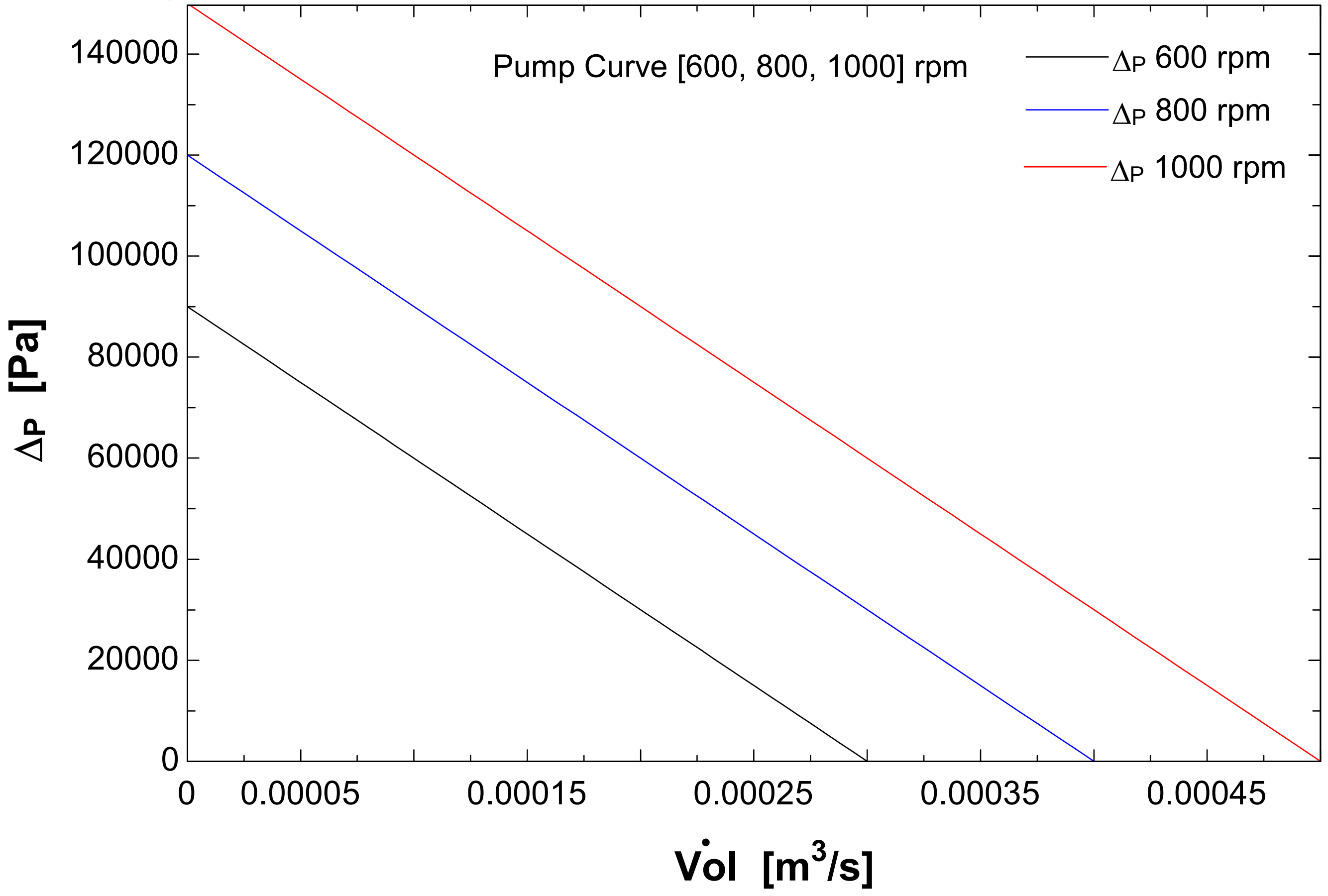
Made with Goodnotes

$\dot{S}_{gen,h}$ = 20.43 [W/K] *Part f/o) Entropy generated in the water heater.*
$\dot{W}_{lost}$ = 6083 [W] {6.083 [kW]} *Part g/p) Total lost work in heater and pump.*
$cost_{tot}$ = 0.4074 [$/hr] *Part h/q) Total cost per hour to run the pool heating system.*
$Frac_{cost,h}$ = 0.9916 [-] *Part i) Fraction of cost associated with running the water heater.*

Arrays Table: Main

	T_i [K] {[F]}	P_i [Pa] {[atm]}	u_i [J/kg]	h_i [J/kg]	$h_{s,i}$ [J/kg]
1	299.8 {80}	101353 {1}	1.259E+06	1.259E+06	
2	299.9 {80.2}	174506 {1.722}	1.260E+06	1.260E+06	1.259E+06
3	299.9 {80.2}	174506 {1.722}	1.260E+06	1.260E+06	
4	314.3 {106.1}	101353 {1}	1.320E+06	1.320E+06	1.317E+06

PART A) PUMP CURVE



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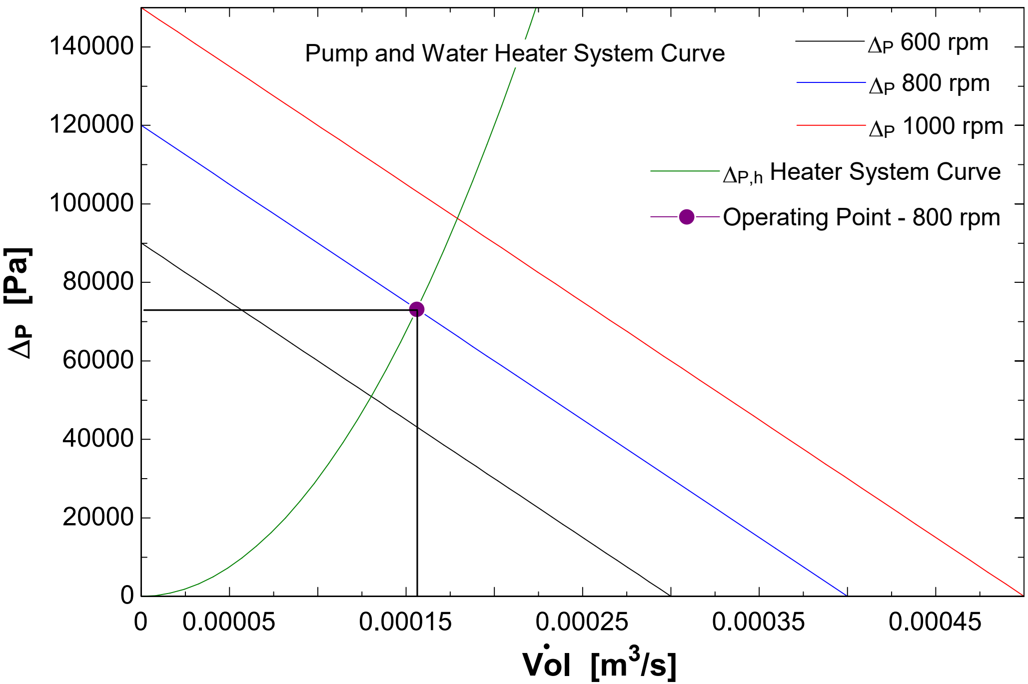
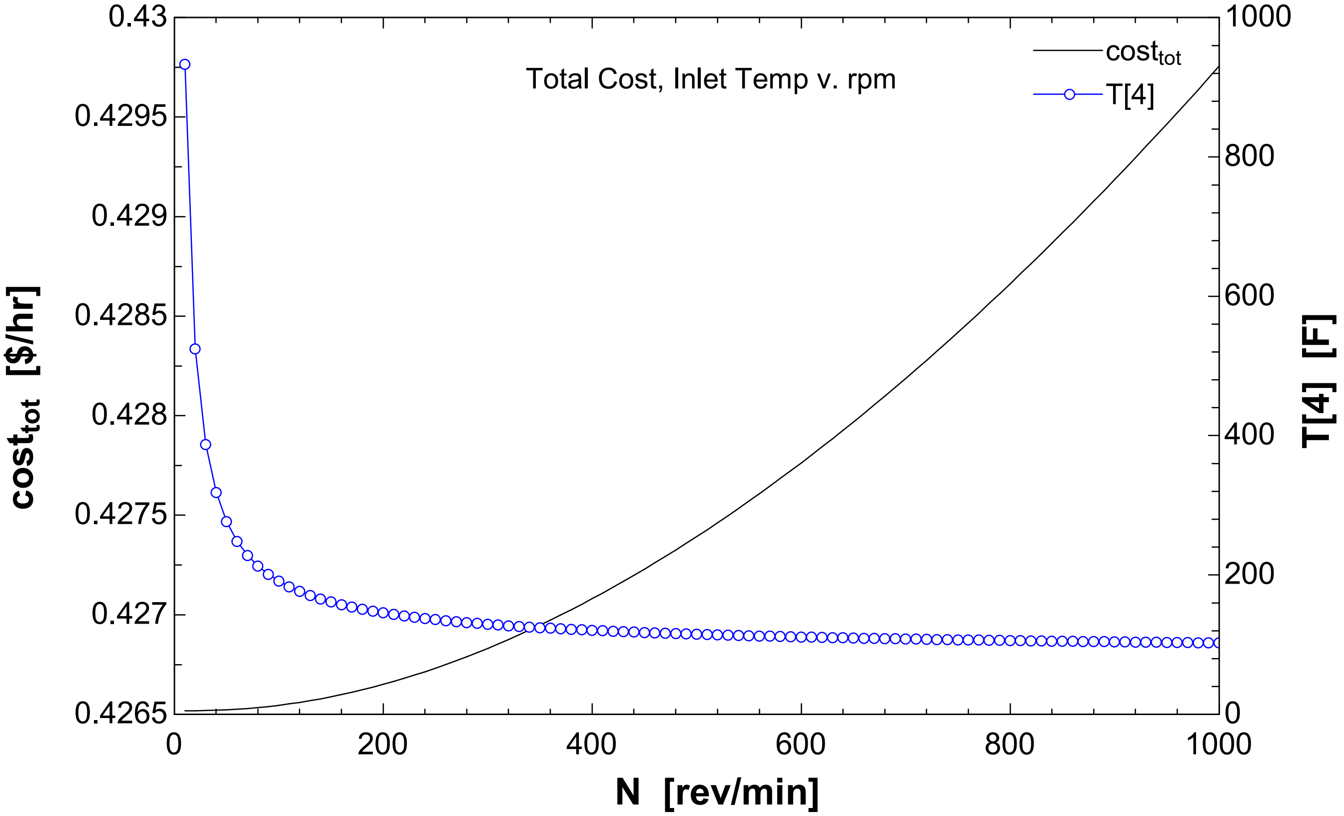
PART B) SYSTEM CURVE OVERLAY



PART J)



EES CODE APPENDIX- DESIGN B (w/ SOLAR COLLECTOR)

```
// Tyler Stevens - ME 461 HW 3 - Cheadle - June 24, 2021
$UnitSystem SI K Pa J mass deg
$Tabstops 0.2 0.4 0.6 0.8 4

////////////////////////////////////////
//Pool Heating System [A]//
////////////////////////////////////////
DESIGN$ = 'B'

"knowns"
eta_p = 0.4 [-]  "pump effieciency"
eta_h = 0.95 [-]                                    "ng heater efficiency"
v_w = 0.001 [m^3/kg]                                "spec vol of pool water"
c_w  =4200 [J/kg-K]                                 "spec heat capacity of water"
P_o = 14.7*convert(psi,Pa)
T_o = converttemp(F,K,70)
T_pool = converttemp(F,K,80)
T_f = converttemp(F,K,1200)
ec = 0.12*convert($/kW-hr,$/W-hr)
ngc = 1.25*convert($/therm,$/W-hr)

////////////////////////////////////////////////////////////////////////////////////
//Modified Pool Heating System - Solar Flux Collector [B]//
////////////////////////////////////////////////////////////////////////////////////
"knowns"
SF = 950 [W/m^2]
A_col = 6 [m^2]
UA = 120 [W/K]
W_dot_sc = SF*A_col

"Pump Curve"
DELTA_P_dh = 150[Pa-min/rev]*N                      "dead head pressure"
Vol_dot_oc = 5E-7[m^3-min/s-rev]*N                  "open circuit flow rate"
DELTA_P = DELTA_P_dh*(1 - (Vol_dot/Vol_dot_oc))     "pump curve"

"System Curves"
DELTA_P_h = 3E12[Pa-s^2/m^6]*Vol_dot^2              "Water Heater System Curve"
DELTA_P_col = 5E12[Pa-s^2/m^6]*Vol_dot^2            "Solar Flux Collector System Curve"
DELTA_P_sys = DELTA_P_h + DELTA_P_col               "Combined Systems Curve Pressure Loss"

"Fixed Parameters"
m_dot = Vol_dot/v_w
// mass balance
{mass flow rate constant for all states: m = m4 = m3 = m2 = m1}

"! State 4-1 Pool"
Q_dot_pool = 9.5*convert(kW,W)
// energy balance
m_dot*h[4] = Q_dot_pool + m_dot*h[1]
// entropy balance
S_dot_gen_pool = (Q_dot_pool/T_pool) + m_dot*c_w*ln(T[1]/T[4])

"! State 1 - Pump Inlet"
T[1] = T_pool
P[1] = P_o
u[1] = c_w*T[1]
h[1] = u[1] + v_w*P[1]

"! State 1-2 Water Pump"
eta_p=W_dot_ps/W_dot_p
// energy balance
m_dot*h[1] + W_dot_p = m_dot*h[2]
// entropy balance
DELTA_s_21 = c_w*ln(T[2]/T[1])
```

Made with Goodnotes

```
S_dot_gen_p = m_dot*DELTA_s_21
// reversible energy balance
m_dot*h[1] + W_dot_ps = m_dot*h_s[2]
h_s[2] = c_w*T[1] + v_w*P[2]

"! State 2 - Pump Outlet"
P[2] = P[1] + DELTA_P
h[2] = c_w*T[2] - v_w*P[2]
u[2] = c_w*T[2]

"! State 3-4 Water Heater"
eta_h = Q_dot_fs/Q_dot_f
// energy balance
m_dot*h[3] + Q_dot_f = m_dot*h[4]
h[4] = c_w*T[4] + v_w*P[4]
// entropy balance
S_dot_gen_h = m_dot*DELTA_s_43 - (Q_dot_f/T_f)
DELTA_s_43 = c_w*ln(T[4]/T[3])
// reversible energy balance
m_dot*h[3] + Q_dot_fs = m_dot*h_s[4]
//h_s[4] = c_w*T[3] + v_w*P[4]

"! State 4 - Heater Outlet"
P[4] = P[3] - DELTA_P_h
u[4] = c_w*T[4]

$IF DESIGN$ = 'A'
    "Part a - Pump Curve at different speeds"
    $IFNOT PARAMETRICTABLE
        Vol_dot = 0.000156155 [m^3/s]
        N = 800 [rev/min]                                       "pump speed"
        //DELTA_P = DELTA_P_h
    $ENDIF
    "Part c - Determine the volumetric flow rate of water throughout the system"
    $IF PARAMETRICTABLE
        DELTA_P = DELTA_P_h                                     "pressure differential req to overcome pressure
loss in heater"
    $ENDIF

    "! State 2-3 Piping"
    // mass balance
    {mass flow rate constant for all states: m3 = m2}

    "! State 3 - Heater Inlet"
    T[3] = T[2]
    P[3] = P[2]
    h[3] = h[2]
    u[3] = u[2]

    "Total Entropy Gen and Lost Work to heater and pump"
    S_dot_gen_hp = S_dot_gen_p + S_dot_gen_h
    W_dot_lost = S_dot_gen_hp*T_o
    "Total cost to run the pool heating system [$/hr]"
    cost_p = ec*W_dot_p
    cost_h = ngc*Q_dot_f
    cost_tot = cost_p + cost_h
    "Fraction of cost associated with running water heater"
    Frac_cost_h = cost_h/(cost_h+cost_p)
$ENDIF

$IF DESIGN$ = 'B'
    $IFNOT PARAMETRICTABLE
        Vol_dot = 0.0001051516 [m^3/s]
        N = 800 [rev/min]                                       "pump speed"
```

Made with Goodnotes

```
      //DELTA_P = DELTA_P_sys                          "pressure differential req to overcome pressure
loss in heater"
   $ENDIF
   "Part c - Determine the volumetric flow rate of water throughout the system"
   $IF PARAMETRICTABLE
      DELTA_P = DELTA_P_sys                            "pressure differential req to overcome pressure
loss in heater"
   $ENDIF

   "! State 2-3 Piping"
   // energy balance
   m_dot*h[2] + W_dot_sc = Q_dot_loss + m_dot*h[3]
   Q_dot_loss = UA*(T[3] - T_o)
   h[3] = c_w*T[3] + v_w*P[3]
   // entropy balance
   S_dot_gen_sf = Q_dot_loss/T[3] + (m_dot*(c_w*ln(T[3]/T[2])))

   "! State 3 - Heater Inlet"
   P[3] = P[2] - DELTA_P_col
   u[3] = c_w*T[3]

   "Fraction of Pool Heating Load delivered by water heater and solar collector"
   Frac_wh_load = Q_dot_f/Q_dot_pool
   Frac_sc_load = (W_dot_sc - Q_dot_loss)/Q_dot_pool

   "Total Entropy Gen and Lost Work to heater and pump"
   S_dot_gen_hp = S_dot_gen_p + S_dot_gen_h
   W_dot_lost = S_dot_gen_hp*T_o
   "Total cost to run the pool heating system [$/hr]"
   cost_p = ec*W_dot_p
   cost_h = ngc*Q_dot_f
   cost_tot = cost_p + cost_h
   "Fraction of cost associated with running water heater"
   Frac_cost_h = cost_h/(cost_h+cost_p)
$ENDIF
```

SOLUTION

Unit Settings: SI K Pa J mass deg

A_{col} = 6 [m^2]
$cost_h$ = 0.2356 [$/hr]
$cost_p$ = 0.00279 [$/hr]
$cost_{tot}$ = 0.2384 [$/hr]
c_w = 4200 [J/kg-K]
Δ_P = 88455 [Pa]
$\Delta_{P,col}$ = 55284 [Pa]
$\Delta_{P,dh}$ = 120000 [Pa]
$\Delta_{P,h}$ = 33171 [Pa]
$\Delta_{P,sys}$ = 88455 [Pa]
$\Delta_{s,21}$ = 1.708 [J/kg-K]
$\Delta_{s,43}$ = 166.8 [J/kg-K]
DESIGN$ = 'B'
ec = 0.00012 [$/W-hr]
η_h = 0.95 [-]
η_p = 0.4 [-]
$Frac_{cost,h}$ = 0.9883 [-]
$Frac_{sc,load}$ = 0.4162
$Frac_{wh,load}$ = 0.5814
$\dot{m}$ = 0.1052 [kg/s]
N = 800 [rev/min]
ngc = 0.00004265 [$/W-hr]
P_o = 101353 [Pa]
$\dot{Q}_f$ = 5523 [W]
$\dot{Q}_{fs}$ = 5247 [W]
$\dot{Q}_{loss}$ = 1746 [W]
$\dot{Q}_{pool}$ = 9500 [W]
SF = 950 [W/m^2]
$\dot{S}_{gen,h}$ = 11.55 [W/K]
$\dot{S}_{gen,hp}$ = 11.73 [W/K]
$\dot{S}_{gen,p}$ = 0.1796 [W/K]
$\dot{S}_{gen,pool}$ = 1.085 [W/K]
$\dot{S}_{gen,sf}$ = 18.53 [W/K]
T_f = 922 [K]
T_o = 294.3 [K]
T_{pool} = 299.8 [K]
UA = 120 [W/K]
$\dot{Vol}$ = 0.0001052 [m^3/s]
$\dot{Vol}_{oc}$ = 0.0004 [m^3/s]
v_w = 0.001 [m^3/kg]
$\dot{W}_{lost}$ = 3452 [W] {3.452 [kW]}
$\dot{W}_p$ = 23.25 [W]
$\dot{W}_{ps}$ = 9.301 [W]
$\dot{W}_{sc}$ = 5700 [W]

No unit problems were detected.

SOLUTION – DESIGN B (SOLAR)

KEY VARIABLES
$\dot{Vol}$ = 0.0001052 [m^3/s] *Part c/k) Volumetric Flow Rate of water pumped through the system.*
$\dot{W}_p$ = 23.25 [W] *Part e/m) Pump Power*

Made with Goodnotes

$Frac_{wh,load}$ = 0.5814 *Part n) Fraction of total pool load delivered by water heater.*

$Frac_{sc,load}$ = 0.4162 *Part n) Fraction of total pool load delivered by solar collector.*

$\dot{S}_{gen,p}$ = 0.1796 [W/K] *Part f/o) Entropy generated in the pump.*

$\dot{S}_{gen,h}$ = 11.55 [W/K] *Part f/o) Entropy generated in the water heater.*

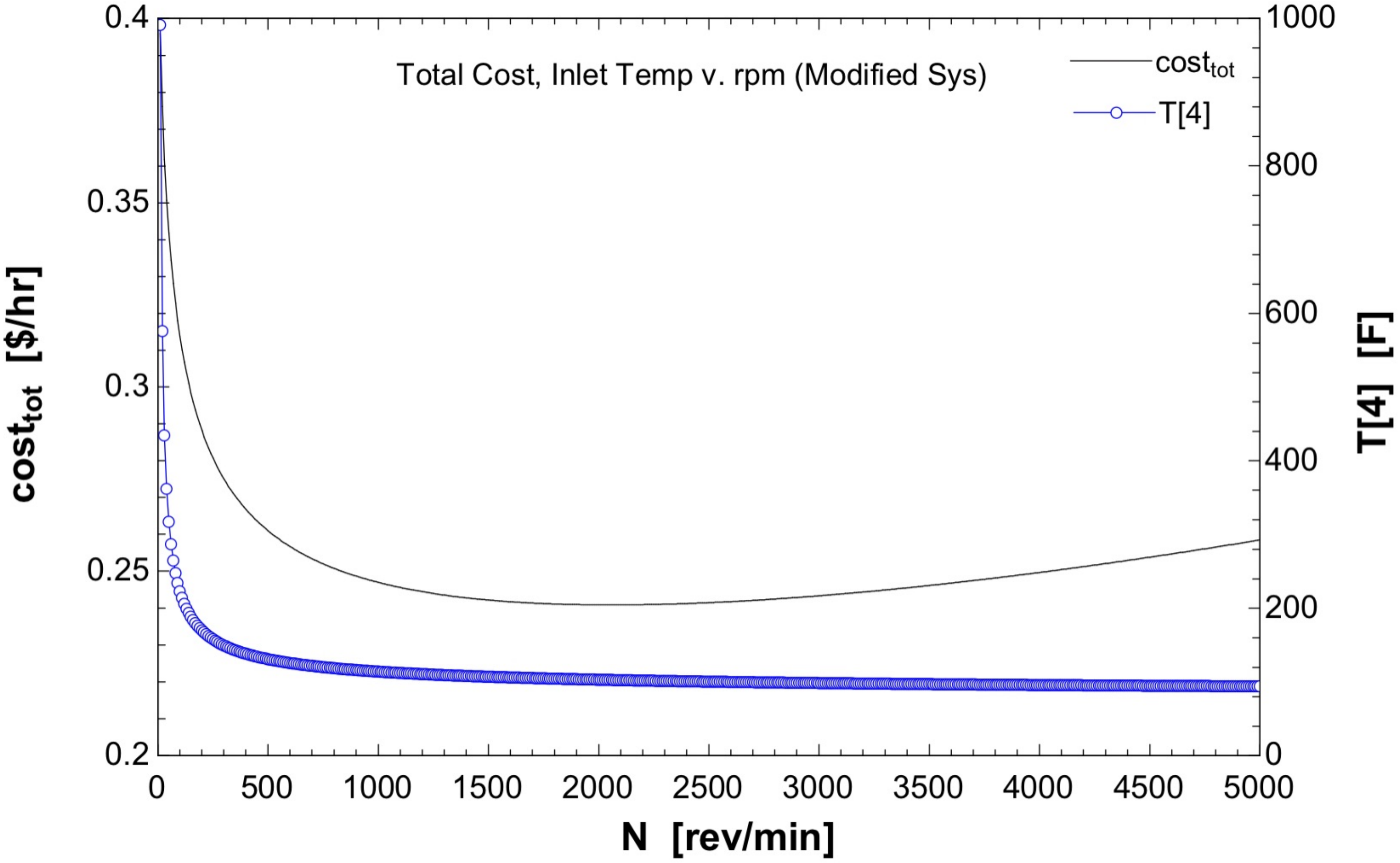
$\dot{W}_{lost}$ = 3452 [W] {3.452 [kW]} *Part g/p) Total lost work in heater and pump.*

$cost_{tot}$ = 0.2384 [$/hr] *Part h/q) Total cost per hour to run the pool heating system.*

Arrays Table: Main

	T_i [K] {[F]}	P_i [Pa] {[atm]}	u_i [J/kg]	h_i [J/kg]	$h_{s,i}$ [J/kg]
1	299.8 {80}	101353 {1}	1.259E+06	1.259E+06	
2	299.9 {80.22}	189807 {1.873}	1.260E+06	1.260E+06	1.259E+06
3	308.8 {96.19}	134523 {1.328}	1.297E+06	1.297E+06	
4	321.3 {118.7}	101353 {1}	1.350E+06	1.350E+06	1.347E+06

PART R)



Made with Goodnotes